EDUCATIONAL WEBSITES TO TEACH RELATIVISTIC DYNAMICS AT HIGH SCHOOL AND INTRODUCTORY UNIVERSITY LEVEL

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Abstract

This research analyzes different educational websites to teach the relativistic dynamics at high school and university level. The results show that sites developed by formal educational institutions such as ministries of education or universities organize knowledge of relativistic dynamics in a similar way to textbooks used in physics teaching. The main way of addressing the relativistic dynamics of websites aimed at high school is based on the concepts of mass and energy, while university sites use the momentum concept.

Key-words: Relativistic dynamics; Websites; Special Theory of Relativity; Textualization; Teaching.

Résumé

Cette recherche analyse différents sites web éducatifs pour enseigner la dynamique relativiste au niveau secondaire et universitaire. Les résultats montrent que les sites développés par des établissements d'enseignement formel tels que les ministères de l'éducation ou les universités organisent la connaissance de la dynamique relativiste d'une manière similaire aux manuels utilisés dans l'enseignement de la physique. La principale façon d'aborder la dynamique relativiste des sites web destinés aux écoles secondaires est basée sur les concepts de masse et d'énergie, tandis que les sites universitaires utilisent le concept de momentum.

Mots-clés : Dynamique relativiste ; Sites web ; Théorie de la relativité restreinte ; Textualisation¸ Enseignement.

Introduction

According with Chevallard &Bosch (2014) the process of didactic transposition refers to the transformations a body of knowledge undergoes from the moment it is produced, put into use, selected, and designed to be taught until it is actually taught in a given educational institution. This process involves both when textbooks or other types of didactic devices, such as websites, are created. In the digital society, students and teachers increasingly turn to Internet 4.0, where they find educational websites.

Research on teaching special relativity focuses more on the notions of relativistic kinematics (loss of simultaneity, length contraction, and time dilation) than on the notions of relativistic dynamics (energy, mass, and momentum). Despite efforts, in introductory high school and university courses, students still do not understand the meaning, origin, and utility of one of the most famous and socially widespread equations of relativistic physics relating energy and mass $E = m.c^2$.

This work is part of a larger investigation (Otero, et. al., 2016), which aims to teach relativistic kinematics and dynamics in the last year of high school. In a previous research (González, et. al., 2022) we analyzed how textbooks used in high school and introductory university courses propose the central notions of relativistic dynamics. Here, we present a research whose objective is to analyze the knowledge transformations proposed on educational websites related to relativistic dynamics and how this knowledge is proposed and formulated.

Textbooks and websites in transpositive processes

The knowledge that is taught in schools, the educational practices involved, and the body of content that students are expected to learn originate from what the Theory of Didactic Transposition calls academic knowledge. This knowledge is generally produced in scientific institutions, including universities, and often integrates elements from various related social practices (Chevallard & Bosch, 2014).

In physics, academic knowledge of Special Relativity Theory (SRT) has been developed through various mathematical formulations that have evolved since Einstein's original proposal. Einstein himself made several efforts to communicate the theory and its implications using non-mathematical language, emphasizing physical concepts and empirical evidence. Today, the academic understanding of relativistic dynamics is grounded in principles of symmetry and conservation, with key concepts such as energy and momentum expressed using four-vectors and tensors in Minkowski spacetime. This formalism is comprehensively presented in Landau and Lifshitz's:The Classical Theory of Fields (Landau, 1980).

A major challenge in teaching relativity arises from the fact that such mathematical tools are not accessible in secondary education or in most introductory university courses. This requires a transformation of academic knowledge, often using historical perspectives or simplified formulations to introduce relativistic concepts in a way that is didactically appropriate. It is important to note, as Chevallard points out, that academic knowledge is neither rigid nor monolithic, and the transposition process is bidirectional across all phases.

Educational websites, textbooks, and other instructional resources play a central role in this transformation. They mediate between academic knowledge, curriculum guidelines, and classroom practice. Therefore, analyzing how academic content is transformed in these materials is essential to understanding its impact on what is actually taught and learned in schools (Chevallard, 1985).

In the case of SRT, we initially analyzed how this transformation occurs in textbooks at various educational levels. In this study, we extend that analysis to online educational resources aimed at communicating this body of knowledge. This examination provides valuable input for the design of a teaching sequence for secondary education, aimed at supporting students' conceptual understanding of SRT by integrating both the key ideas of relativistic kinematics and dynamics.

Research questions

1. What are the notions of relativistic dynamics proposed on websites for high school and introductory university courses, and what are their differences?
2. What similarities and differences can be identified between the knowledge proposed on websites and the knowledge previously analyzed in textbooks?

Methodology

The relativistic dynamics sections of 15 educational websites, selected through purposive sampling, were analyzed. Only websites that aim to teach relativistic dynamics at the high school and university levels were included. As we just have pointed out below, this article focuses exclusively on the analysis of websites. The comparison with textbooks is made only through reference to our prior work (González et al., 2022).
The sites were analyzed based on an inductive categorization, generating nominal variables and their respective modalities that arise from the considered corpus. These variables can be included within two metacategories called *general characteristics* and *knowledge transformations*, this can be seen in detail in the Annex (Table 1). Some of these variables coincide with those identified in the analysis focused on textbooks (González, et. al., 2022), allowing to describe the similarities and differences between the considered websites and textbooks.

Data Analysis and discussion

Approximately half of all reviewed websites are associated with educational institution. In the context of university websites, the remaining half are attributed to independent authors. In contrast, among high school websites, a quarter of them lack identifiable authors and references to content sources, while the other quarter is credited to independent authors. In terms of navigability, the more attractive websites, characterized by well-organized information presentation, are predominantly found within the university-level category, while half of the high school websites are disordered and offer a less attractive browsing experience. This is shown in the Figure 1.

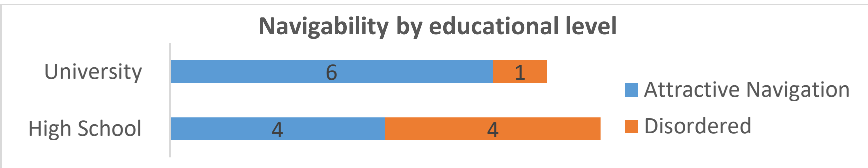

Figure 1 - Navigability according to educational level

Regarding the use of audiovisual resources, for both educational levels, videos and simulations are little used. Only one website features videos and two sites feature simulations. Furthermore, at both educational levels, written communication is prioritized and very few images are used.

In terms of the gateway to relativistic dynamics, as illustrated in Figure 2, more than half of the university websites focus on the concept of Mass, while the rest focus on Momentum ($\vec{p} =$

$m.\vec{v}$), and none on Energy. In contrast, high school websites are evenly split between exploring the concepts of Mass and Energy, with only a few addressing the concept of Momentum.

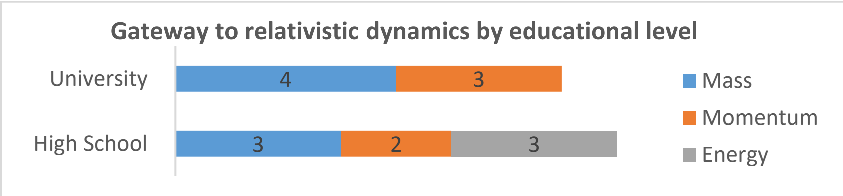

Figure 2 – Gateway to the relativistic dynamics according to educational level.

Figure 3 shows that most university websites propose a questioning of classical dynamics concepts due to their incompatibility with conservation laws at high speeds. In contrast, high school-oriented websites are mainly based on the invalidity of Newton's second law in the relativistic limit. Notably, the Figure 3 indicates that nearly a quarter of high school websites do not provide a rationale for introducing relativistic dynamics, nor do they distinguish between these two models.

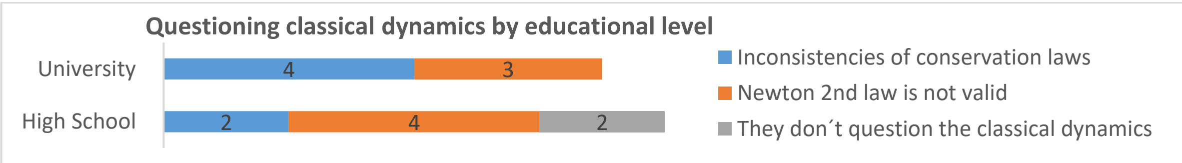

Figure 3 - Questioning classical dynamics according to educational level.

On the other hand, we notice from Figure 4 an absence of collision analysis at high school level. In contrast, at three university websites employed one-dimensional and two-dimensional collisions, with a preponderance of the two-dimensional case. Furthermore, there is no significant difference between both educational levels about empirical references and applications of relativistic dynamics. At both levels, there are few empirical references and applications, as Figure 5 shows. Furthermore, the examples presented are predominantly linked to scientific research rather than everyday applications familiar to the reader.

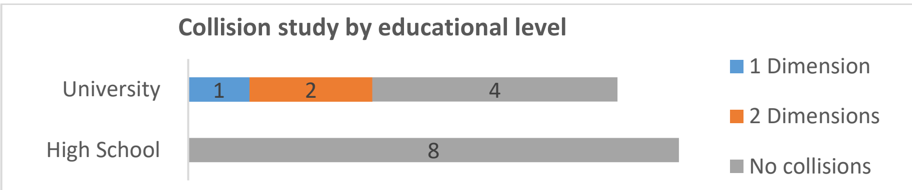

Figure 4 – Analysis of collision study according to educational level.

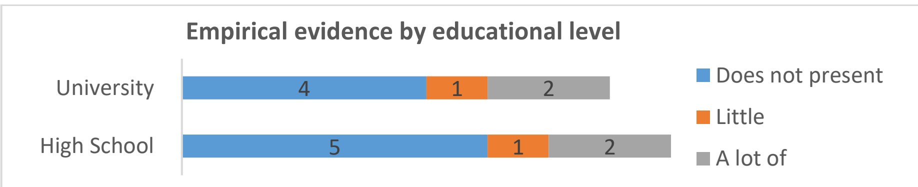

Figure 5 – Presence of empirical evidence according to educational level.

The individual analysis of the core concepts of relativistic dynamics reveals significant differences based on the educational level. When examining the momentum concept, the arguments provided by university and high school websites exhibit notable disparities, as illustrated in Figure 6. University websites commonly argue for the modification of momentum, either due to the modification of the concept of mass or because classical momentum is not conserved at high speeds. In contrast, most high school websites do not give importance to momentum, and it remains an unaddressed topic.

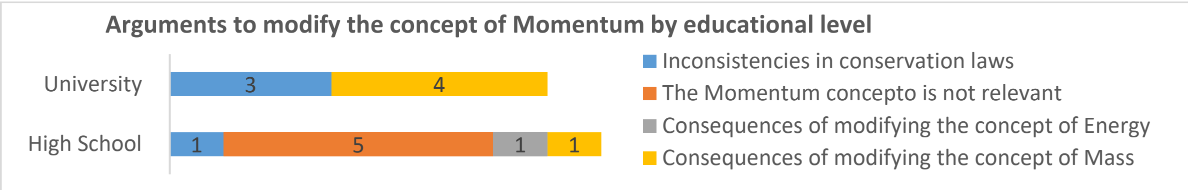

Figure 6 - Argument to modify the classic concept of Momentum according to educational level.

On the other hand, as shown in Figure 7, it is evident that both university and high school websites exhibit similar patterns when it comes to modify the Mass concept. Approximately half of them postulate that mass increases with speed. However, almost half of the university websites contend that the alteration in the mass concept is a consequence of the changes in the momentum concept within the STR.

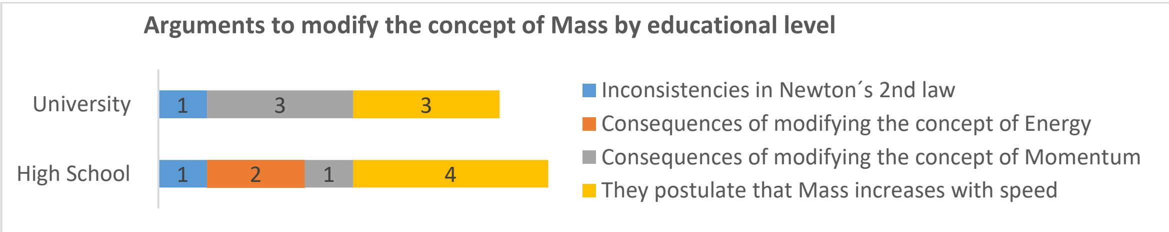

Figure 7 - Argument to modify the classic concept of Mass according to educational level.

Regarding the concept of energy, for both educational levels, approximately one third of the websites refer to Einstein's work and postulate the equation that introduces relativistic energy, this is shown in the Figure 8. Nearly half of university websites, on the other hand, underpin the introduction of the energy concept through the alteration of the mass concept. In the case of high school websites, a quarter of them present the same argument, while the rest do not address the concept of energy, or rely on inconsistencies in the laws of conservation, or the modification of momentum.

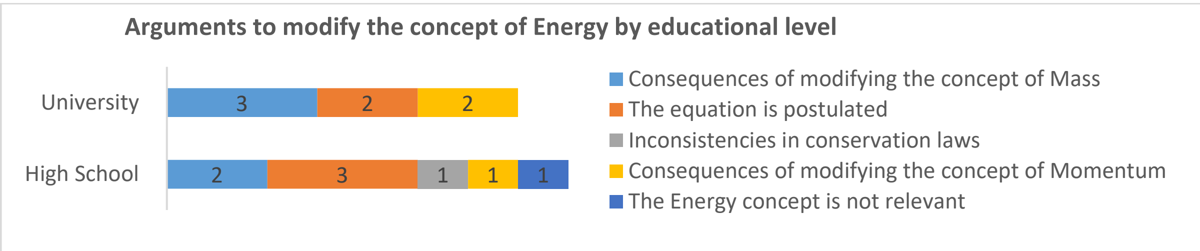

Figure 8 - Argument to modify the classic concept of Energy according to educational level.

Finally, if we focus on the transformations of knowledge and compare them according to the authorship, we can see in the Figure 9 that in websites linked to educational institutions the concept of Momentum as a gateway to relativistic dynamics predominates. However, sites not linked to a specific institution are based on the concept of Mass and Energy.

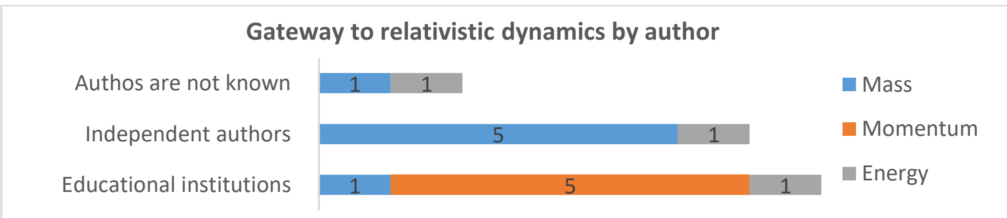

Figure 9 – Gateway to relativistic dynamics according to authorship.

Analysis of main ways of approaching relativistic dynamics

Comparing the different categories related to *knowledge transformation*, a general scheme (Figure 10) emerges, depicting the main ways taken by websites based on their approach to Special Theory of Relativity dynamics and the motivations driving its inclusion.

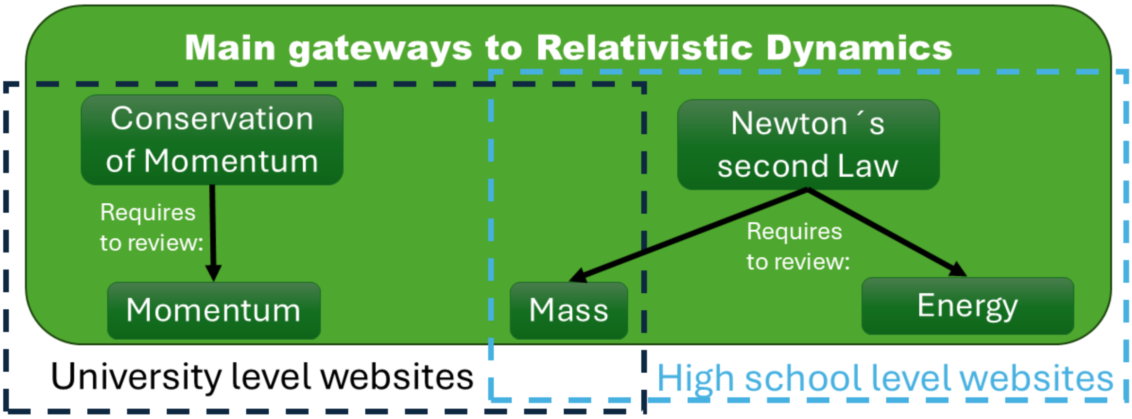

Figure 10 – Main ways of approaching relativistic dynamics.

In general, websites introduce STR for two primary reasons. The first reason is that classical momentum is not conserved at high speeds, necessitating a re-evaluation of this concept. This way is shown on the left side of Figure 10 and is used by approximately half of the university websites. When comparing the data from categories *gateway to relativistic dynamics* and *questioning classical dynamics*, it is evident that more than three quarters of the websites that use the Momentum concept as gateway specifically employ this argument for introducing relativistic dynamics.

The second argument is that Newton's second law is no longer valid at high speeds, requiring a revision of associated concepts. This way is shown on the right side of Figure 10 and is used by approximately half of the university sites and the majority of the high school websites. Similarly, upon comparing categories *gateway to relativistic dynamics* and *questioning classical dynamics*, it is apparent that approximately three quarter of the websites that use the Mass and Energy concept as a gateway, rely on this argument to introduce STR dynamics. In some cases, this is achieved through thought experiments or experimental evidence illustrating the impossibility of accelerating an object beyond the speed of light *c*.

On the other hand, when comparing category *gateway to relativistic dynamics* with categories *arguments to modify the concept of Mass* and *Energy*, it becomes evident that most websites that introduce the Momentum concept, argue that changes in the concepts of Mass and Energy stem from this modification. Furthermore, when comparing category *gateway to relativistic dynamics* with categories *arguments to modify the concept of Mass* and *Energy*, it is apparent that most websites introducing the Mass concept argue that changes in the concepts of Momentum and Energy result from the revision of the Mass concept. Finally, when comparing category *gateway to relativistic dynamics* with categories *arguments to modify the concept of Mass* and *Momentum*, we observe that most websites introducing the Energy concept argue that the Mass concept changes as a result of this. However, many of these websites do not place significant importance on Momentum and do not incorporate it into relativistic dynamics.

Sites developed by formal educational institutions show similarities with textbooks in most categories: section format, main approaches to relativistic dynamics, arguments, and questioning of classical dynamics (González et. al., 2022). They are also similar in the importance they attribute to experiments and applications of relativistic dynamics.

Consequently, most of the sites developed by educational institutions analyzed here transfer knowledge from textbooks and adapt it to the digital format, often neglecting the simulation and visualization possibilities offered by websites.

Conclusions

The results indicate that educational websites addressing concepts of relativistic dynamics tailor their mathematical formulations to the intended educational level, whether for introductory university courses or secondary education. These resources typically adopt a conceptual approach, deliberately minimizing formal mathematical structures. This represents a significant departure from academic formulations, which often employ advanced mathematical frameworks. However, as previously discussed, even within academic contexts, one can observe transpositive phenomena, beginning with Einstein's original 1905 presentation of Special Relativity Theory (SRT), which emphasized conceptual understanding over formalism.

Websites mainly address the concepts of momentum, mass and energy to teach relativistic dynamics. There are differences between the approach adopted by websites aimed at high school and university level. High school level sites introduce relativistic dynamics by questioning Newton's laws, mainly mass and energy. University level websites question the conservation of mass or momentum and omit energy. High school websites avoid collision analysis, because momentum is not in the curriculum. Although the momentum-based approach is epistemologically more appropriate, it is not viable in high school because classical collisions are not studied there.

There are notable differences in the use of audiovisual resources between institutional websites and the rest. In most cases, websites tend to use images in a traditional way and refrain from incorporating videos or simulators. Thus, only certain institutional sites use them, perhaps because they require a competent and dedicated team of designers, not always accessible to independent authors.

In both types of sites there are few references to experimental evidence, to the detriment of students' interest in relativistic dynamics and its implications, such as in nuclear energy, its medical and military applications, which would lead to sizing the scope of the equation $E = mc^2$.

The under-utilization of resources such as videos, simulations and images are common to websites at both educational levels. To such an extent that websites designed for high school students would be less attractive than those designed for university students. Finally, given the scarce presence of STR in the curriculum and the divergence of approaches adopted, we highlight the importance of didactic research dedicated to the teaching of relativistic dynamics that leads to the development of appropriate educational resources.

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Annex

Table 1 – Nominal variables with their respective modalities

General characteristics	
Educational level	High School
	University
Type of Resource	Web pages
	Websites
Authorship	Educational institutions are responsible
	Independent authors
	Authors are not known
Navigability	Attractive navigation
	Disordered
Use of images	Traditional
	Introductory
	Imagistic
Use of simulations	The use of simulations is central
	Complementary use of simulations
	They don´t use simulations
Use of videos	The use of videos is central
	They don´t use videos
Knowledge transformations	
Gateway to relativistic dynamics	Mass
	Momentum
	Energy
Questioning classical dynamics	Inconsistencies of conservation laws at high speeds
	Newton's 2nd law is not valid at high speeds
	They do not question the classical dynamics
Collision study	One-dimensional collisions
	Two-dimensional collisions
	They don´t study collisions
Significance attributed to experiments	Doesn´t present experimental evidence
	Little experimental evidence
	A lot of experimental evidence
Applications of relativistic dynamics	Focused on scientific research
	Focused on everyday applications
	Both
Arguments to modify the concept of Momentum	Inconsistencies in conservation laws
	The Momentum concept is not relevant
	Consequences of modifying the concept of Energy
	Consequences of modifying the concept of Mass
Arguments to modify the concept of Mass	Inconsistencies in Newton's 2nd Law
	Consequences of modifying the concept of Energy
	Consequences of modifying the concept of Momentum
	They postulate that mass increases with speed
Arguments to modify the concept of Energy	Consequences of modifying the concept of Mass
	The equation is postulated
	Inconsistencies in conservation laws
	Consequences of modifying the concept of Momentum
	The Energy concept is not relevant